WEST BENGAL STATE UNIVERSITY

B.Sc. Honours Part-III Examination, 2020

PHYSICS

PAPER-PHSA-V

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

UNIT-VA

1. Answer any ***five*** questions from the following: 3×5=15

(a) What do you mean by constraints? What is the type of constraint in case of pendulum with length varying with time?

(b) What is meant by canonical transformation?

(c) What is meant by proper time?

(d) Draw the world line of a particle moving with speed 2×10^8 m/s along x-axis.

(e) Two electrons move towards each other with speed 0.9c. Calculate the relative speed of one with respect to another.

(f) State the postulate of equal a priori probability.

(g) Draw the allowed phase space for a one dimensional linear harmonic oscillator of mass 'm', vibrating with frequency 'ω' and energy ranging from 0 to E.

(h) What is Fermi momentum? Why is it non-zero even at $T = 0$?

GROUP-A

Answer any *one* question from the following

2. (a) A particle is constrained to be in a plane. It is subjected to a force directed to a fixed point P on the plane and is inversely proportional to the square of the distance from P. 2+2+2

(i) Using polar coordinates, write the Lagrangian of this particle.

(ii) Write the Euler-Lagrange equation.

(iii) Show that angular momentum of the particle is conserved.

(b) Using Legendre transformation construct the Hamiltonian function from Lagrangian. Now find the Hamilton's equations. 2+2

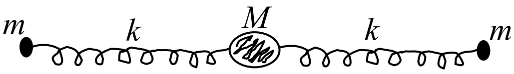
3. (a) Show that the transformation given by $Q=\sqrt{2q}\,e^{a}\cos p$, $P=\sqrt{2q}\,e^{-a}\sin p$ is canonical. 2

(b) From Poisson Bracket relation $\{q_i,\ p_j\}=\delta_{ij}$, show that $\{L_x,\ L_y\}=L_z$. 3

(c) Consider the longitudinal motion of the system of masses and springs with $M>m$. 1+4

(i) Write down the Lagrangian of the system.

(ii) What are the normal mode frequencies of the system?



OR

GROUP-B

Answer any *one* question from the following

4. (a) Write down the Lorentz transformation equations between two inertial frames moving relative to each other with a velocity v along common X-axis. 2

(b) Show that two successive Lorentz transformation with velocities v_1 and v_2 in the same direction are equivalent to a single Lorentz transformation with a velocity $v = \dfrac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{c^2}}$. 4

(c) What are (i) space-like (ii) time-like and (iii) light-like interval? Is it possible to transform a time-like vector into a space-like one? 3+1

5. (a) Explain the phenomena ‘Length contraction’ using Lorentz transformation equations. 3

(b) Derive the relativistic expression for the kinetic energy of a particle. Show that it reduces to the expression $\frac{1}{2}mv^2$ if $v << c$. 3

(c) A body of mass m at rest breaks up spontaneously into two parts having rest mass m_1 and m_2 and respective speeds v_1 and v_2. Using conservation of mass-energy show that $m > m_1 + m_2$. 4

OR

GROUP-C

Answer any *one* question from the following

6. (a) A classical particle is free to move in a cube of side L having its energy lying between E and $E + \Delta E$. Find the number of microstates available to it. 3

(b) If three identical particles are distributed over three single particle states how many possibilities are allowed if the particles are (i) Bosons and (ii) Fermions. 2+2

(c) Find the partition function of an ideal monatomic gas. 3

7. (a) Sketch the FD-distribution function at the absolute zero of temperature and finite non-zero temperature. 2

(b) Show that average energy at $T = 0$ is $\varepsilon_{av} = \frac{3}{5} \in_{\text{F}} (0)$, where $\in_{\text{F}} (0)$ is the Fermi energy at $T = 0$. 4

(c) (i) Show that average energy $\overline{E} = -\dfrac{\partial}{\partial \beta}(\ln Z)$ where $z = \sum_r e^{-\beta E_r}$ is the partition function. 4

(ii) Obtain an expression for $\overline{(\Delta E)^2} = \overline{E^2} - \overline{E}^2$. Show that $\overline{(\Delta E)^2} = \dfrac{\partial^2}{\partial \beta^2}(\ln Z)$.

8. (a) Derive Bose-Einstein distribution function stating clearly the assumptions. 4

(b) Establish Planck’s radiation law for a photon gas obeying B.E. statistics. 4

(c) What is Bose condensation? 2

UNIT-VB

9. Answer any ***five*** questions from the following: 3×5=15

(a) What is the de-Broglie wave associated with an electron having kinetic energy 100 eV?

(b) What are the properties of a 'well-behaved' wave function?

(c) What do you mean by a stationary state in quantum mechanics?

(d) Using the vector atom model, determine the possible values of total angular momentum of an *f*-electron.

(e) Show that for a given principal quantum number *n*, maximum number of possible electrons is $2n^2$.

(f) Define the expectation value of a dynamical quantity.

(g) Angular part of the wave-function associated with a particle is given by $\psi(\theta, \phi) = \frac{1}{\sqrt{3}}(\sqrt{2}Y_{11} - Y_{10})$, where Y_{lm}'s represents spherical harmonics. A measurement of $\hat{L}_2$ on the state is followed by another measurement of $\hat{L}_2$. Find the probability of getting $L_2 = 1$ in the first and second measurements.

(h) Show that the spin magnetic moment of electron is equal to the Bohr magneton.

GROUP-D

Answer any *one* question from the following

10.(a) Using Ehrenfest's theorem show that the expectation value of the position of a particle moving in three dimensions with the Hamiltonian $H = \frac{\vec{p}^2}{2m} + V(\vec{r})$ satisfies $\frac{d}{dt}\langle \vec{r} \rangle = \frac{\langle \vec{p} \rangle}{m}$. 6

(b) Consider a particle that moves in one dimension. Two of its normalized energy eigenfunctions are $\psi_1(x)$ and $\psi_2(x)$ with energy eigenvalues E_1 and E_2. At $t = 0$ the wave function for the particle is 2+2

$$\phi = c_1\psi_1(x) + c_2\psi_2(x) \text{ where } c_1,\ c_2 \text{ constants}$$

(i) Find the wave function $\phi(x, t)$ as a function of time, in terms of the given constants and initial condition.

(ii) Find an expression for the expectation value of the particle position $\langle x \rangle$ as a function of time for the state $\phi(x, t)$ from part (i).

11.(a) State the orthonormality condition of two wave functions. 2

(b) Calculate the normalisation constant for a wave function (at $t = 0$) given by $\psi(x) = ae^{-\alpha^2 x^2/2} e^{ikx}$. Determine (i) the probability density and (ii) probability current density. 2+(1+2)

(c) A one dimensional wavefunction is given by $\psi(x) = \sqrt{\alpha} e^{-\alpha x}$. Find the probability of finding the particle between $x = \frac{1}{a}$ and $x = \frac{2}{a}$. 3

12.(a) An electron is confined in a one dimensional box of length L. What should be the length of the box to make its zero-point energy is equal to its rest mass energy (m_0c^2)? Express the result in terms of Compton wavelength. 2

(b) If there is a two level system with energy eigenvalues E_1 and E_2 with corresponding eigenstates ϕ_1 and ϕ_2 respectively and the system is in a state ψ such that the probability of getting each of the energy value on measurement is equal to that of the other, find the time dependence of $|\psi|^2$. 3

(c) Show that for all the energy eigenstates in a harmonic oscillator $\langle x \rangle$ vanishes though $\langle x^2 \rangle$ does not. 3

(d) At time $t = 0$ the wavefunction of the hydrogen atom is prepared as 2

$$\psi(\vec{r}, t, 0) = \frac{1}{\sqrt{10}}(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1})$$

where the subscripts are values for the quantum numbers (n, l, m). Find the expectation value for the energy of the system.

13.(a) Show that for Hydrogen atom problem $[\hat{\text{H}}, \hat{L}^2] = 0$. 2

(b) Using the relation $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, prove that $[\hat{L}_x, \hat{L}_z] = -i\hbar\hat{L}_y$ and $[\hat{L}_y, \hat{L}_z] = i\hbar\hat{L}_x$. 3

(c) If $\hat{L}_+ = \hat{L}_x + i\hat{L}_y$, using previous relations, find $[\hat{L}_z, \hat{L}_+]$. If ϕ_m is an eigenfunction of $\hat{L}_z$ with eigenvalue $m\hbar$, prove that $\hat{L}_+\phi_m$ is also another eigenfunction of $\hat{L}_z$ with eigenvalue $(m+1)\hbar$. 1+2

(d) Like H-atom, positronium is a bound state of an electron and a positron. Ground state energy of positronium is a factor '*f*' times that of an H-atom. Find the value of '*f*'. 2

OR

GROUP-E

Answer any *one* question from the following

14.(a) What is Normal Zeeman effect? Show with the diagram the longitudinal and transverse views of Normal Zeeman effect. 1+2

(b) What is anomalous Zeeman effect? Obtain an expression for Lande g factor from it. 3

(c) Draw Zeeman splittings of the D_2 and D_1 lines of sodium corresponding to transitions from the excited states $3\,^2P_{3/2}$ and $3\,^2P_{1/2}$ to the ground state $3\,^2S_{1/2}$. 4

15.(a) Show the vibrational and rotational energy levels of a diatomic molecule on a potential energy versus inter-atomic distance curve. Explain the formation of these levels. 2+2

(b) Explain the physical reason behind the more pronounced deviation of higher vibrational levels in the case of a diatomic molecules from Harmonic Oscillator levels. 2

(c) State Hund's rule for multi-electron atoms. 2

(d) The spectroscopic term of the ground state of last unfilled subshell of an atom is ^{5}D. Find the total spin quantum number S and total orbital angular momentum quantum number L. 2

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