WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 4th Semester Examination, 2020

MTMACOR09T-MATHEMATICS (CC9)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the closure of $\{(x,y):\ x^2+y^2\leq 1\}$.

(b) Check whether $S=\{(0,1)\}$ is open or closed in $\mathbb{R}^2$.

(c) Show that $f(x,y)=|x|+|y|$ is not differentiable at (0, 0).

(d) If $u=f(x^2+2yz,\ y^2+2zx)$ then prove that

$$(y^2-zx)\frac{\partial u}{\partial x}+(x^2-yz)\frac{\partial u}{\partial y}+(z^2-xy)\frac{\partial u}{\partial z}=0.$$

(e) Show that the function $f(x,y)=2x^4-3x^2y+y^2$ has neither a maximum nor a minimum at (0, 0).

(f) Evaluate $\int_C (y^2dx-x^2dy)$ along the straight line joining (0, 1) and (1, 0).

(g) Find the work done in moving a particle in the force field $\boldsymbol{F}=(3x^2,\,2xz-y,\,z)$ along the straight line joining (0, 0, 0) and (2, 1, 3).

(h) Check whether the vector field given by $\boldsymbol{F}=(y^2+z^3,\,2xy-5z,\,3xz^2-5y)$ is conservative or not.

2. (a) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of that box which requires least material for construction. 4+4

(b) Let f, a function of two variables x and y be continuous at an interior point (a,b) of its domain of definition, and $f(a,b)\neq 0$. Show that there exists a neighbourhood of (a,b) in which $f(x,y)$ retains the same sign as that of $f(a,\ b)$.

3. (a) A function $f(x,y)$ is defined as: 4+4

$$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}}; & (x,y)\neq(0,0)\\ 0\quad ; & (x,y)=(0,0)\end{cases}$$

Show that f is continuous but not differentiable at (0, 0).

(b) Check whether $\lim\limits_{(x,y)\to(0,0)} \frac{|x|}{y^2} e^{-|x|/y^2}$ exists or not.

4. (a) Evaluate $\iint\limits_R (x+y)\,dxdy$ over the rectangle $R=[0,1;\ 0,2]$. 4+4

(b) Prove that $f(x,y)=\{|x+y|+(x+y)\}^k$ is everywhere differentiable for all values of $k \geq 0$.

5. (a) Let $f: S \to \mathbb{R}$ be a function where $S \subset \mathbb{R}^2$. If f is continuous at a point $(a,b) \in S$, then show that $f(x,b)$ is continuous at $x=a$ and $f(a,y)$ is continuous at $y=b$. Is the converse true? Justify your answer. (2+2) + (1+1+1+1)

(b) $f(x,y)$ is defined as:

$$f(x,y)=\begin{cases} x\sin\frac{1}{x}+y\sin\frac{1}{y} & ; \quad xy \neq 0 \\ 0 & ; \quad xy=0 \end{cases}$$

Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists but the repeated limits do not exist. Is $f(x,y)$ continuous at (0, 0)?

6. (a) By changing the order of integration prove that 4+4

$$\int_0^1 dx \int_x^{1/x} \frac{y^2 dy}{(x+y)^2\sqrt{1+y^2}} = \frac{1}{2}(2\sqrt{2}-1)$$

(b) If a differentiable function $f(x,y)$ of two variables x and y when expressed in terms of new variables u and v defined by $x=\frac{u+v}{2}$ and $y=\sqrt{uv}$ becomes $g(u,v)$, then show that

$$\frac{\partial^2 g}{\partial u \partial v} = \frac{1}{4}\left(\frac{\partial^2 f}{\partial x^2} + \frac{2x}{y}\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y^2} + \frac{1}{y}\frac{\partial f}{\partial y}\right)$$

7. (a) If $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$, show that a stationary value of $a^3x^2+b^3y^2+c^3z^2$ is given by $ax=by=cz$, and this gives an extreme value if $abc(a+b+c)$ is positive. 4+4

(b) Find the volume common to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ and the cylinder $x^2+y^2=ay$.

8. (a) Use Stokes' theorem to prove that $\text{div}(\text{curl } \vec{F})=0$ and $\text{curl}(\text{grad } \phi)=\vec{0}$. Where $\vec{F}(x,y,z)$ is a vector function and $\phi(x,y,z)$ is a scalar function. 4+4

(b) Evaluate $\oint\limits_C [(1-x^2)ydx+(1+y^2)xdy]$, where C is $x^2+y^2=a^2$.

9. (a) Find the surface area of the sphere $x^2+y^2+z^2=9$ lying inside the cylinder $x^2+y^2=3y$. 4+4

(b) Use divergence theorem to evaluate

$$\iint_S (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy)$$

where S is the closed surface bounded by the planes $z = 0$, $z = b$ and the cylinder $x^2 + y^2 = a^2$.

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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